KÄHLER POTENTIALS FOR ORBIFOLDS
WITH CONTINUOUS WILSON LINES AND
THE SYMMETRIES OF THE STRING ACTION

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ABSTRACT

By employing the symmetries of the underlying conformal field theory, the tree-level Kähler potentials for untwisted moduli of the heterotic string compactifications on orbifolds with continuous Wilson lines are derived. These symmetries act linearly on bosonic (toroidal and $E_8 \times E_8$ gauge) string coordinates as well as on the untwisted (toroidal and continuous Wilson lines) moduli; they correspond to the scaling of toroidal moduli, the axionic shift of toroidal moduli and the shift of the continuous Wilson line moduli. In turn such symmetries provide sufficient constraints to determine the form of the low-energy effective action associated with the untwisted moduli up to a multiplicative factor.

The determination of a low-energy four-dimensional (4-d) lagrangian is a necessary step towards making phenomenological predictions from heterotic string theory, such as the study of gauge coupling unification, the masses of quarks and leptons, supersymmetry breaking and string cosmology. In particular, heterotic string theories compactified on orbifolds are of phenomenological importance as they give rise to an $N = 1$ space-time supersymmetric semi-realistic 4-d quantum field theories [1, 2]. The orbifold models are characterized by a set of continuous parameters describing the size and shape of the orbifold known as toroidal moduli. They are the marginal deformations of the underlying conformal field theory of the orbifold [3] and they enter in the space-time $N = 1$ supersymmetric four dimensional Lagrangian as chiral fields with flat potentials to all orders in perturbation theory. Toroidal moduli belong to the untwisted sector of the orbifold. On the other hand, the blowing-up modes, which parameterize the resolution of the orbifold conical singularities belong to the twisted sector of the orbifold.

An appealing feature of orbifold compactification is that the toroidal moduli can be included explicitly in the calculations of the low-energy effective action.[*] thus allowing for probing the features of the effective action in the whole sector of toroidal moduli. The method to determine the explicit structure of the superpotential of orbifold compactification directly from the underlying conformal field theory was given in [5]. The explicit dependence on toroidal moduli has been calculated for Yukawa couplings[†] and non-renormalizable terms [7] in the superpotential. The explicit dependence of the tree level Kähler potential on the toroidal moduli has been evaluated in the literature by several methods. One method relies on the truncation of the field theory limit of the string [8, 9]. Another method [10, 11] employs the symmetries of the world-sheet action. A similar method has also been used in Ref. [12]. In addition, significant progress has been made in addressing toroidal moduli dependence for genus-one (threshold) corrections[‡] to

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[*] In contrast, the explicit dependence on the blowing-up modes can be addressed only perturbatively in the value of the blowing-up modes [4].

[†] See for example Ref. [6] and references therein.

[‡] See for example [13] and references therein.

the low-energy action, in particular to the gauge and gravitational couplings.

In addition to the toroidal moduli, the untwisted sector of the heterotic string theory compactified on orbifolds also contains Wilson line moduli, which arise when the twist defining the orbifold is realized on the $E_8 \times E_8$ root lattice by a rotation [14, 15]. The inclusion of Wilson line moduli is essential as they lead to models with semi-realistic gauge groups (of lower rank) via the "stringy Higgs effect" [16], and thus may provide a class of orbifold compactifications with a semi-realistic features.

Recently, there has been a renewed interest in addressing the low-energy effective lagrangian for orbifolds with continuous Wilson lines [17, 18, 19, 20]. As a first step one would like to address the explicit dependence of the Kähler potential on the Wilson line moduli, thus allowing for the full description of the untwisted sector of the orbifold moduli space. In Ref. [17] the local structure of the moduli space of $\mathbf{Z}_N$ orbifolds with continuous Wilson lines was obtained, by using the compatibility between the Narain twist and the toroidal moduli, *i.e.*, one starts with the moduli space of toroidal compactification [21] and then determine which subspace is compatible with the action of the orbifold twist on the underlying Narain lattice.

The purpose of this letter is to derive the explicit dependence of the Kähler potentials on the full untwisted sector of the orbifolds with continuous Wilson lines by using the symmetries of the the underlying conformal field theory which is associated with the full untwisted moduli sector of the corresponding orbifold compactification. This method provides a generalization of the one in [10,11] which only addresses the Kähler potential for toroidal moduli and untwisted matter fields.

We start our discussion with the heterotic string theory compactified on a torus. A d-dimensional torus ($d = 6$) can be defined as a quotient of $\mathbf{R}^d$ with respect to a lattice Λ defined by

$$\Lambda = \{\sum_{i=1}^{d} a^i \mathrm{e}_i, \quad a^i \in Z\}, \qquad i = 1, \cdots d \ . \tag{1}$$

The world-sheet action associated with the bosonic string coordinates is of the

form [22]:

$$S = \frac{1}{2\pi} \int dz d\bar{z} \left(b_{ij} \partial \phi^j \bar{\partial} \phi^i + A_{Ij} \partial \phi^j \bar{\partial} \varphi^I + C_{IJ} \partial \varphi^J \bar{\partial} \varphi^I \right) , \tag{2}$$

supplemented with the constraint $C_{IJ} \partial \varphi^J + A_{Ij} \partial \phi^j = 0$. Here, z ($\bar{z}$) and ∂ ($\bar{\partial}$) correspond to the left (right)-moving world-sheet coordinates and the corresponding partial derivative, respectively, b_{ij} is the background metric denoting the metric and the antisymmetric tensor coefficients (toroidal moduli), A_{Ij} is the Wilson line and C_{IJ} is the Cartan metric of $E_8 \times E_8$. Denote the internal toroidal string coordinates by the d-dimensional column matrix ϕ and the $E_8 \times E_8$ bosonic string coordinates by the D-dimensional ($D = 16$) column matrix by φ. The equations of motion plus the chiral constraint can be written in matrix notation in the following form:

$$\bar{\partial} G = 0, \qquad G = bF + A^t C \mathcal{F} \tag{3}$$

$$\mathcal{G} = C\mathcal{F} + CAF = 0 , \tag{4}$$

where $F = \{\partial \phi^j\}$, $\mathcal{F} = \{\partial \varphi^J\}$, b is a $d \times d$ matrix representing the metric and antisymmetric tensor background fields b_{ij}, A is $D \times d$ matrix representing the Wilson line moduli $A^I{}_j$, and C is the Cartan matrix (C_{IJ}) of $E_8 \times E_8$.

The equations of motion have the following symmetries. First, there is a rescaling symmetry given as

$$\begin{aligned}
b &\to \left(L^{-1}\right)^t b L^{-1}, \\
A &\to AL^{-1}, \\
F &\to LF, \\
\mathcal{F} &\to \mathcal{F},
\end{aligned} \tag{5}$$

where L is an invertible constant $n \times n$ matrix.

Second, there is the well known symmetry under the axionic shift:

$$b \rightarrow b + N \ ,$$
$$A \rightarrow A \ ,$$
$$F \rightarrow F \ ,$$
$$\mathcal{F} \rightarrow \mathcal{F},$$

(6)

where $N = -N^t$. The symmetries (5) and (6) are those discussed in Ref. [10] and are used to fix the form of the kinetic energy term of the b moduli up to an overall constant, which is determined to be 1 (in Planck units) by calculating the string scattering amplitude with four toroidal moduli vertex insertions. Consequently, the form of the corresponding Kähler potential is fully determined.

In our case, an additional symmetry, which relates the toroidal and Wilson line moduli, is needed. It is obtained by imposing a constant shift W on the Wilson line moduli A, along with the following transformation on the toroidal moduli b as well as on the bosonic string coordinates:

$$A \rightarrow A + W \ ,$$
$$b \rightarrow b + W^t C A + A^t C W + W^t C W \ ,$$
$$F \rightarrow F \ ,$$
$$\mathcal{F} \rightarrow \mathcal{F} - W F \ .$$

(7)

The constraint equation (4) is crucial to prove that the equation of motion (3) is invariant under the symmetry transformations (7).[⋆]

The above sets of symmetry transformations (5)-(7) correspond to the symmetries of the underlying conformal field theory and therefore correspond to the symmetry for the corresponding low-energy effective action associated with the untwisted moduli sector. These symmetries are sufficient to determine the tree-level

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⋆ Note, the symmetry transfomations (5)-(7) are linear in the background fields.

kinetic terms of the untwisted moduli sector up to an overall constant. Namely, the kinetic energy of of the untwisted sector moduli is of the form:

$$K = \frac{\partial^2 K}{\partial T_i \partial T_j} \partial_\mu T_i \partial^\mu T_j \quad , \tag{8}$$

where T_i correspond to the untwisted moduli fields in b and A, where K is, up to an multiplicative constant, of the following form:

$$K = -\log \det(b + b^t - 2A^t C A) \quad . \tag{9}$$

The multiplicative constant can be determined by a calculation of the string amplitude with four toroidal moduli vertex insertions.

The strucutre of the coset space associated with the metric $\frac{\partial^2 K}{\partial T_i \partial T_j}$ (defined in (8)-(9)) is of the type $SO(d, d+D)/SO(d) \otimes SO(d+D)$ $(d = 6, \ D = 16)$.

We would also like to point out that the equations of motion (3)-(4) actually possess a larger symmetry [12] than the one specified by symmetry transformations (5)-(7). The larger symmetry corresponds in general to a subgroup of $SO(d+D, d+D)$ $(d = 6, \ D = 16)$ which is consistent with the constraints $\mathcal{G} = 0$ (Eq. (4)). If one represents an element of $SO(d+D, d+D)$ with

$$g = \begin{pmatrix} g_1 & g_2 \\ g_3 & g_4 \end{pmatrix} , \qquad g^t \Omega g = \Omega , \qquad \Omega = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \tag{10}$$

then the group element has the following action on the moduli (background) fields and the corresponding bosonic string coordinates:

$$\begin{pmatrix} F \\ \mathcal{F} \\ G \\ \mathcal{G} \end{pmatrix} \rightarrow g \begin{pmatrix} F \\ \mathcal{F} \\ G \\ \mathcal{G} \end{pmatrix} . \tag{11}$$

The symmetry transformations (5)-(7) correspond to a subset of those specified in

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Eq. (11), and thus, can be obtained by setting:

$$g_1 = \begin{pmatrix} \alpha_1 & 0 \\ \alpha_2 & 1 \end{pmatrix}, \qquad g_2 = 0, \qquad g_3 = \begin{pmatrix} \gamma_1 & 0 \\ 0 & 0 \end{pmatrix}, \qquad \text{with } \gamma_1 = -\gamma_1^t . \qquad (12)$$

Using (10) and (12) we obtain the following explicit form of the subset of the group elements:

$$g = \begin{pmatrix} \alpha_1 & 0 & 0 & 0 \\ \alpha_2 & 1 & 0 & 0 \\ \gamma_1 & 0 & (\alpha_1^{-1})^t & -(\alpha_1^{-1})^t \alpha_2^t \\ 0 & 0 & 0 & 1 \end{pmatrix}, \qquad (13)$$

which in turn transforms for the untwisted moduli in the following way

$$b \to \gamma_1 \alpha_1^{-1} + (\alpha_1^{-1})^t b \alpha_1^{-1} - (\alpha_1^{-1})^t (\alpha_2)^t C A \alpha_1^{-1} - (\alpha_1^{-1})^t A^t C \alpha_2 \alpha_1^{-1}$$

$$+ (\alpha_1^{-1})^t \alpha_2^t C \alpha_2 \alpha_1^{-1} , \qquad (14)$$

$$A \to A \alpha_1^{-1} - \alpha_2 \alpha_1^{-1} .$$

Obviously, these transformations constitute the rescaling (5), axionic shift (6), and the shift of the Wilson lines (7).

The analysis for the toroidal case can now be generalized in a straightforward way to the case of orbifolds with continuous Wilson lines. A symmetric d-dimensional($d = 6$) $\mathbf{Z}_N$ orbifold [1] can be obtained by identifying points on the $\mathbf{T}^d$ torus under the action of a cyclic group $\mathbf{Z}_N$, generated by the twist θ,

$$\mathbf{Z}_N = \{\theta^j, j = 1, ..., N - 1\} . \qquad (15)$$

In order to allow for the presence of continuous Wilson line moduli, the twist should also act as a rotation on the $E_8 \times E_8$ lattice coordinates. Let the action of the twist on the toroidal string coordinates be represented by $(d \times d)$ matrix Q and

on the gauge coordinates by the $(D' \times D')$ matrix M, with D' the dimension of subspace of the $E_8 \times E_8$ lattice on which the twist has non-trivial action. Then for the untwisted moduli to be consistent with the action of the twist, they have to satisfy the following conditions [18, 19]:

$$Q^t b Q = b , \qquad MA = AQ \quad . \tag{16}$$

Due to the above conditions, the orbifold has fewer moduli than the corresponding torus and thus the string action for the untwisted sector of the $\mathbf{Z}_N$ orbifolds contains the subset of terms in the toroidal string action (2)(supplemented by the chiral constraint (4)), which are invariant under the corresponding twist transformations. Thus, the symmetry transformations of the underlying conformal field theory constitute a subset of transformations (5)-(7), now acting on the remaining moduli and the string coordinates on which the twist has a non-trivial action. Consequently, the effective 4-dimensional kinetic terms of the moduli fields and the corresponding Kähler potential of all $\mathbf{Z}_N$ orbifolds can be determined explicitly. The coset spaces spanned by the untwisted moduli of $\mathbf{Z}_N$ orbifolds correspond to Kähler spaces, which are subspaces of $\dfrac{SO(d, d+D)}{SO(d) \otimes SO(d+D)}$ $(d = 6, \ D = 16)$.

For the discussion of orbifolds it is in certain cases more convenient to represent the d-toroidal (ϕ^j) and D-gauge (φ^I) real string coordinates in a complex basis by Z^a and $\mathcal{Z}^m$ string coordinates, respectively. A twist of order N acts on the complex coordinates as a multiplication by $e^{2\pi i/N}$. The complex coordinates are related to the real ones via the relation [23]:

$$
\begin{aligned}
\phi^i &= Z^a E_a^i + \bar{Z}^{a*} E_{a*}^i , & a, a^* &= 1, 2, ..., \frac{d}{2}(= 3) , \\
\varphi^I &= \mathcal{Z}^m \mathcal{E}_m^I + \bar{\mathcal{Z}}^{m*} \mathcal{E}_{m*}^I , & m, m^* &= 1, 2 \cdots, \frac{D'}{2} ,
\end{aligned}
\tag{17}
$$

where E_a, E_{a*} are the complex basis vectors of the orbifold and $\mathcal{E}_m$, $\mathcal{E}_{m*}$ are those of a D'-dimensional $(D' \le D = 16)$ subspace of the $E_8 \times E_8$ lattice. Using the same

symmetries (5)-(7) as before, however, now all expressed in the complex basis, the Kähler potential can be written as

$$K = -\log \det(\mathbf{b} + \mathbf{b}^\dagger - 2\mathbf{A}^\dagger\mathbf{C}\mathbf{A}) \ , \tag{18}$$

where $\mathbf{b}$ and $\mathbf{A}$ are complex matrices representing the background metric and Wilson lines and $\mathbf{C}$ is the Cartan metric expressed in the complex basis.

Thus, the corresponding subsets of symmetry transformations (5)-(7) provide us with the determination of the effective 4-dimensional action, *i.e.*, the kinetic energy, the local structure of the coset spaces spanned by the toroidal and continuous Wilson line moduli of $\mathbf{Z}_N$ - orbifolds. In particular, the untwisted subsector of $\mathbf{Z}_N$ -orbifolds associated with a two-torus ($\mathbf{T}^2$) modded out by a Z_2 twist, has the *same* moduli as the corresponding two-torus, *i.e.*, there are torodial moduli (four real fields) b_{ij} ($(i,j) = 1,2$) and Wilson line moduli whose number depends on the choice of the gauge twist. The corresponding kinetic energy for these moduli is of the type (8)-(9). The coset space spanned by the moduli is the Kähler space [24] of the type $\dfrac{SO(2,r)}{SO(2) \otimes SO(r)}$. The set of complex coordinates for these cosets can be constructed [26, 27, 25, 17] in terms of the real toroidal and Wilson line moduli. The explict transformation between the four real toroidal moduli and four Wilson line moduli, and the corresponding two complex toroidal and two complex Wilson line moduli, representing the coset $\dfrac{SO(2,4)}{SO(2) \otimes SO(4)}$, was given in Ref. [17].[★]

In order to further illustrate the method, we consider a 2-dimensional ($d = 2$) $\mathbf{Z}_3$ orbifold constructed from the torus $\dfrac{R^2}{\Lambda}$, where Λ is the root lattice of $SU(3)$, and for the sake of simplicity we take the gauge twist to act on an $SU(3)$ subgroup of the $E_8 \times E_8$. The action of the twists on the toroidal and the subset of the

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[★] The space with the toroidal moduli b_{ij}, only, corresponds the coset space $\frac{SO(2,2)}{SO(2)\otimes SO(2)} \equiv \left[\frac{SU(1,1)}{U(1)}\right]^2$. The explict transformation between the real fields b_{ij} and the corresponding two-complex toroidal moduli was given in Ref [10]. For related transformations with inclusion of untwisted matter fields see Refs. [26, 27, 25].

$E_8 \times E_8$ bosonic coordinates are represented by:

$$Q = \begin{pmatrix} 0 & -1 \\ 1 & -1 \end{pmatrix} , \quad M = \begin{pmatrix} 0 & -1 \\ 1 & -1 \end{pmatrix} . \tag{19}$$

Because of condition (16) this orbifold has one independent Wilson line modulus [17]. The Cartan matrix C_{IJ} of $SU(3)$ and Wilson line fields $A^I{}_j$ are represented in the real lattice basis by the following matrices:

$$C = \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix} , \quad A = \begin{pmatrix} A^1{}_1 & A^1{}_2 \\ A^2{}_1 & A^2{}_2 \end{pmatrix} . \tag{20}$$

Recall, the lower and upper indices of $A^I{}_j$ refer to the toroidal (target space) and gauge indices, respectively. Using the consistency conditions (16) with the twists represented as (19), the background metric b and A can be represented as

$$A = \begin{pmatrix} p & q \\ -q & p+q \end{pmatrix} , \quad b = \begin{pmatrix} g_{11} & -g_{11}/2 - B \\ -g_{11}/2 + B & g_{11} \end{pmatrix} . \tag{21}$$

The complex basis of the orbifold and the gauge coordinates are given by

$$\begin{aligned} E_1 &= e_1 - \alpha e_2 , \\ \mathcal{E}_1 &= e_1 - \alpha e_2 , \qquad \alpha = e^{2i\pi/3} \end{aligned} \tag{22}$$

The complex coordinates are given in terms of the real coordinates by

$$\begin{aligned} Z^1 &= \frac{\alpha^2}{(\alpha^2 - \alpha)} \phi^1 + \frac{1}{(\alpha^2 - \alpha)} \phi^2 , \\ \mathcal{Z}^1 &= \frac{\alpha^2}{(\alpha^2 - \alpha)} \varphi^1 + \frac{1}{(\alpha^2 - \alpha)} \varphi^2 . \end{aligned} \tag{23}$$

The world-sheet action (for the twisted coordinates) can now be written as

$$S = \frac{1}{2\pi} \int dz d\bar{z} \left(\mathbf{b} \partial Z^1 \bar{\partial} \bar{Z}^{1*} + \mathbf{A} \partial Z^1 \bar{\partial} \bar{\mathcal{Z}}^{1*} + \mathbf{C} \partial \mathcal{Z}^1 \bar{\partial} \bar{\mathcal{Z}}^{1*} \right) + h.c. \quad . \tag{24}$$

Notice that this is the most general world-sheet action consistent with the action of both the toroidal and gauge twists. The complex moduli are expressed in terms

of the real ones by

$$\mathbf{b} = \frac{3}{2}g_{11} + i\sqrt{3}B = \sqrt{3}(\sqrt{\det g} + iB) \equiv \sqrt{3}T \ ,$$

$$\mathbf{A} = 3(p - \alpha q) \equiv \frac{3}{\sqrt{2}}\mathcal{A} \ , \qquad (25)$$

$$\mathbf{C} = 3 \ .$$

Using (18), the Kähler potential can be written, up to an overall positive multiplicative factor, in the following complex form :

$$K = -\log(T + T^* - \sqrt{3}\mathcal{A}^*\mathcal{A}) + const. \ , \qquad (26)$$

This is the Kähler potential for the $\frac{SU(1,2)}{SU(2) \otimes U(1)}$ coset space.

If, for example, one allows the gauge twist to act on another $SU(3)$ subgroup with the eigenvalue $e^{4\pi i/2}$, i.e., in the real basis the twist is generated by $M' = \begin{pmatrix} -1 & 1 \\ -1 & 0 \end{pmatrix}$, then the moduli space will contain another complex Wilson line modulus. Denote $\mathcal{E}_2$ the complex vector representing the second $SU(3)$. In the real basis the Wilson line is given by

$$A = \begin{pmatrix} p & q \\ -q & p+q \\ p' & q' \\ p'+q' & -p' \end{pmatrix} . \qquad (27)$$

The world-sheet action in this case can be written as

$$\mathcal{S} = \frac{1}{2\pi}\int dz d\bar{z}\Big(\mathbf{b}\partial Z^1\bar{\partial}\bar{Z}^{1*} + \mathbf{A}\partial Z^1\bar{\partial}\bar{\mathcal{Z}}^{1*} + \mathbf{C}\partial \mathcal{Z}^1\bar{\partial}\bar{\mathcal{Z}}^{1*}$$
$$+ \mathbf{C}'\partial \mathcal{Z}^2\bar{\partial}\bar{\mathcal{Z}}^2 + \mathbf{A}'\partial Z^1\bar{\partial}\bar{\mathcal{Z}}^2\Big) + h.c. \ , \qquad (28)$$

where $\mathbf{A}' = 3(p' - \alpha q') \equiv \frac{3}{\sqrt{2}}\mathcal{A}'$ and $\mathbf{C}' = 3$. It is then straightforward to show

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that the Kähler potential can be written as

$$K = -\log(T + T^* - \sqrt{3}\mathcal{A}^*\mathcal{A} - \sqrt{3}\mathcal{A}'^*\mathcal{A}') \ . \tag{29}$$

This is the Kähler potential for the $\dfrac{SU(1,3)}{SU(3) \otimes U(1)}$ coset space.

In general, $\dfrac{SU(1,k)}{SU(k) \otimes U(1)}$ coset space is parametrized by one complex modulus T and $k-1$ complex Wilson line moduli. This coset space corresponds to the untwisted moduli sector of any 2-dimensional $\mathbf{Z}_N (N \neq 2)$ orbifold. Therefore, it also corresponds to a subsector of d-dimensional $(d = 6)$ $\mathbf{Z}_N$ orbifold, which corresponds to the untwisted moduli sector of a two-torus modded out by a Z_N twist represented by a complex phase. In models corresponding to the string vacua the number of the complex Wilson lines of course depends on the choice of the gauge twist which is constrained by the world-sheet modular invariance [14, 15]. Applying the method presented to all $\mathbf{Z}_N$ orbifolds, one obtains the same coset spaces as those in [11, 17] with the Wilson lines playing the role similar to those of untwisted matter fields.

In conclusion, we have presented a method to determine the Kähler potential of the moduli fields, *i.e.*, toroidal moduli as well as continuous Wilson lines, in the untwisted sector of the heterotic string theory compactified on any $\mathbf{Z}_N$ orbifold. The method employs the symmetries of the underlying conformal field theory associated with the untwisted moduli sector of the theory. These symmetries act linearly on bosonic (toroidal and $E_8 \times E_8$ gauge) string coordinates as well as the untwisted (toroidal and continuous Wilson lines) moduli; they correspond to the scaling of toroidal moduli, the axionic shift of toroidal moduli and the shift of the continuous Wilson line moduli. Such symmetries provide sufficient constraints to determine the form of the low-energy effective action of the untwisted moduli sector up to a multiplicative factor. In turn, the local structure of the Kähler manifold associated with of untwisted moduli sector can be fully determined.

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